

Some questions that might (not) appear in the exam...

The following questions should give you an impression of what to expect from the exam. Please note that the following questions are not “exam-proof“, i.e., they have not been double-checked for consistency and clarity. Moreover, the questions are not structured into separate problems. You should manage to answer the question within the time indicated in squared brackets.

Assume there are no limits on short sales and the *percentage*¹ returns on two risky assets are bivariate normally distributed and characterized by their vector of expected values, $\boldsymbol{\mu}$, and their covariance matrix, $\boldsymbol{\Sigma}$, as follows:

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 1 & 3 \end{pmatrix}.$$

- a) Compute the equation of the minimum variance set for the two risky assets. [9 minutes]

Hint: $\boldsymbol{\Sigma}^{-1} = \begin{pmatrix} 0.21 & -0.07 \\ -0.07 & 0.35 \end{pmatrix}$.

- b) Compute the weights of the *Global Minimum Variance Portfolio* (GMVP) and the corresponding expected portfolio return and variance. [4 minutes]

- c) Explain the difference between the opportunity set and the minimum variance set [3 minutes].

- d) Assume now that the returns of the assets are uncorrelated. Compute the weight of the first asset for the portfolio with the lowest variance. Explain when the weight of the first asset is larger than 0.5.

Hint: Do not use matrix algebra here but try a direct approach. [5 minutes]

¹That is, $r_{it} = 100 \times \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$, where $P_{i,t}$ is the price of asset i at time t .

- e) Describe the basic idea of the shortfall-based approach and when it might be preferable over the Mean-Variance approach. [4 minutes]
Note: Derivation of the shortfall line not required.
- f) Explain Telser's criterion for determining optimal portfolios. Draw a picture to illustrate the concept. [6 minutes]
- g) Find the optimal Telser-portfolio for a shortfall target of $\tau = 0$ and a shortfall probability of $\alpha = 0.1$. What are the mean and standard deviation of the resulting portfolio? [8 minutes]
- h) Write down and briefly explain the central equation of the zero-beta Capital Asset Pricing Model. [6 minutes]
- i) In the SIM, what is the only reason why stocks vary together, systematically? What is the key assumption about the shocks ϵ_i for this to be the case? [4 minutes]
- j) What is the use of principal components regarding multi-index models? [3 minutes]