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Exercise Sheet 1

1. Monte Carlo Simulation

Show that for any continuous and monotone increasing cumulative distribution function F the random variable defined by $Y := F^{-1}(U)$, $U \sim \mathbb{U}[0,1]$, is distributed according to distribution F. That is,

$$Y \overset{d}{\sim} F.$$

2. VaR, ES: Meaning

You have invested 500,000 € in an investment fonds. The manager of the fonds tells you that the 99% Value-at-Risk for a time horizon of one year amounts to 5% of the portfolio value. Explain the information conveyed by this statement.

The fondsmanager corrects himself. Instead of the Value-at-Risk, the Expected Shortfall amounts to 5% of the portfolio value. How does this statement have to be interpreted? Which of both cases does imply the riskier portfolio?

3. VaR, ES: market risk

After transforming the daily DAX closing values from 01.01.1995 to 12.31.2005 to logarithmic returns, the riskmanagement division of a bank wants to estimate the 99% Value-at-Risk for daily returns associated with an investment in the index. The estimation of $VaR_{0.99}$ shall be based on an approximation of the historical return series by a normal distribution. Using maximum likelihood estimation, the riskmanagement division comes up with estimated parameter values $\hat{\mu} = 0.0344$ and $\hat{\sigma} = 1.5403$. Based on the assumption of iid. normally distributed daily logarithmic returns, calculate $VaR_{0.99}$ and $ES_{0.99}$ for daily returns, as well as for the time horizons of 5 and 10 days, and name two reasons that might lead to poor estimation results associated with the application of this estimated framework.

4. VaR, ES: Multi-period Portfolio loss

The portfolio of an investment fonds consists of d stocks, with number of shares of stock i denoted by λ_i. In order to approximate the distribution of cumulated losses over a period of 3 days, write the 3-day portfolio losses as a function of individual daily logarithmic returns. Then, assume iid. normally distributed daily logarithmic returns and approximate the distribution of losses by linearization of the function. Given this simplifications, does an analytical solution exist? Name all critical assumptions that influence the quality of this approximation.

5. Coherence

Consider a portfolio consisting of $d = 100$ corporate bonds. The probability of default shall be 0.5% for each firm, with occurrence of default independently of each other. Given no default occurs, the value of the associated bond increases from $x_t = 100$ € this year to $x_{t+1} = 102$ € next year, while the value decreases to 0 in the event of default.

Calculate $VaR_{0.99}$ for a portfolio A consisting of 100 shares of one given corporate, as well as for a portfolio B, which consists of one shares of each of the 100 different corporate bonds. Interpret the results. What does that mean for VaR as a risk measure, and what can be said about Expected Shortfall with regard to this feature?