Riskmanagement introduction - summary

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prices vs returns

Why don't we model prices P_t as random variables instead of returns?

prices vs returns

prices have changing mean value over time as they tend to increase on average $\Rightarrow$ no stationarity

log returns

Why are logarithmic returns used so commonly?

log returns

domain changes

$]-\inf, \inf[$ instead of $]0, \inf[$ or $]-1, \inf[$ $\Rightarrow$ could make finding suitable distribution easier

aggregation over time

aggregation over time through summation:

$$r_{1:2}^{log} = r_1^{log} + r_2^{log}$$

consequences of summation

What are consequences from summation over time?

consequences of summation

theoretical justification of normal distribution

$r_t^{log} \sim \mathcal{N}$ could be justified through central limit theorem

analytical solutions

distribution of $r_{1:2}^{log}$ could be solved analytically for well behaved distributions

consequences of modelling returns

What are consequences of modelling returns instead of prices?

consequences of modelling returns

some quantities of interest are functions of random variables:

$$Z := absoluteLoss = f(r_t^{log})$$

What is the distribution of *absoluteLoss*? $\Rightarrow$ transformation theorem

problems with functions of random variables

What problems could occur when applying functions to random variables, even if calculation of pdf may be easily possible through transformation theorem?
What makes linear functions special?

problems with functions of random variables

Usually we are interested in measures that reduce information contained in distribution. These measures often involve integration:

$$\mathbb{E}[Z], \mathbb{V}(Z)$$

may not be solved analytically.
However: possible for linear functions!

interpretation of log returns

Although being just some mathematical transformation to achieve mathematical tractability, why do we often not bother to translate results back to discrete returns?

interpretation of log returns

logarithmic returns are very close to net discrete returns for small returns and hence retain an easy interpretation:

$$r^{log} = \log(1 + r^{net}) \approx r^{net}$$

mean log return

With observed prices P_t and P_{t+n}, how do we get mean log return?

mean log return

mean log return is

$$\frac{1}{n}(\log(P_{t+n}) - \log(P_t))$$

differences in price evolution

What differences exist between the evolution of discrete prices and logarithmic prices?

differences in price evolution

Exponential increase $\leftrightarrow$ linear increase