Univariate Time Series Analysis

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SS 2016

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Introduction

Time series analysis:

- Focus: Univariate Time Series and Multivariate Time Series Analysis.
- A lot of theory and many empirical applications with real data
- Organization:
  - 12.04. - 24.05.: Univariate Time Series Analysis, *six* lectures (Klaus Wohlrabe)
  - 31.05. - End of Semester: Multivariate Time Series Analysis (Stefan Mittnik)
  - 15.04. - 27.05. mondays and fridays: Tutorials (Univariate): Malte Kurz, Elisabeth Heller
- $\Rightarrow$ **Lectures and Tutorials are complementary!**

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Tutorials and Script

- Script is available at: *moodle* website (see course website)
- Password: armaxgarchx
- Script is available at the day before the lecture (noon)
- All datasets and programme codes
- Tutorial: Mixture between theory and R - Examples

Literature

- **Shumway and Stoffer (2010): Time Series Analysis and Its Applications: With R Examples**
- Box, Jenkins, Reinsel (2008): Time Series Analysis: Forecasting and Control
- Lütkepohl (2005): Applied Time Series Econometrics.
- Hamilton (1994): Time Series Analysis.
- Lütkepohl (2006): New Introduction to Multiple Time Series Analysis
- Chatham (2003): The Analysis of Time Series: An Introduction
- Neusser (2010): Zeitreihenanalyse in den Wirtschaftswissenschaften

Examination

- Evidence of academic achievements: Two hour written exam both for the univariate and multivariate part
- Schedule for the Univariate Exam: 30/05 (to be confirmed!!!)

Prerequisites

- Basic Knowledge (ideas) of OLS, maximum likelihood estimation, heteroscedasticity, autocorrelation.
- Some algebra

Software

Where you have to pay:

- STATA
- Eviews
- Matlab (Student version available, about 80 Euro)

Free software:

- R (www.r-project.org)
- Jmulti (www.jmulti) (Based on the book by Lütkepohl (2005))

Tools used in this lecture

- usual standard lecture (as you might expected)
- derivations using the whiteboard (not available in the script!)
- live demonstrations (examples) using Excel, Matlab, Eviews, Stata and JMulti
- live programming using Matlab

Outline

- Introduction
- Linear Models
- Modeling ARIMA Processes: The Box-Jenkins Approach
- Prediction (Forecasting)
- Nonstationarity (Unit Roots)
- Financial Time Series

Goals

After the lecture you should be able to ...

- ... identify time series characteristics and dynamics
- ... build a time series model
- ... estimate a model
- ... check a model
- ... do forecasts
- ... understand financial time series

Questions to keep in mind

General Question	Follow-up Questions
	All types of data
How are the variables defined?	What are the units of measurement? Do the data comprise a sample? Ifo so, how was the sample drawn?
What is the relationship between the data and the phenomenon of interest?	Are the variables direct measurements of the phenomenon of interest, proxies, correlates, etc.?
Who compiled the data?	Is the data provider unbiased? Does the provider possess the skills and resources to enure data quality and integrity?
What processes generated the data?	What theory or theories can account for the relationships between the variables in the data?
	Time Series data
What is the frequency of measurement	Are the variables measured hourly, daily monthly, etc.? How are gaps in the data (for example, weekends and holidays) handled?
What is the type of measurement?	Are the data a snapshot at a point in time, an average over time, a cumulative value over time, etc.?
Are the data seasonally adjusted?	If so, what is the adjustment method? Does this method introduce artifacts in the reported series?

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Goals and methods of time series analysis

The following section partly draws upon Levine, Stephan, Krehbiel, and Berenson (2002), *Statistics for Managers*.

Goals and methods of time series analysis

- understanding time series characteristics and dynamics
- necessity of (economic) forecasts (for policy)
- time series decomposition (trends vs. cycle)
- smoothing of time series (filtering out noise)
  - moving averages
  - exponential smoothing

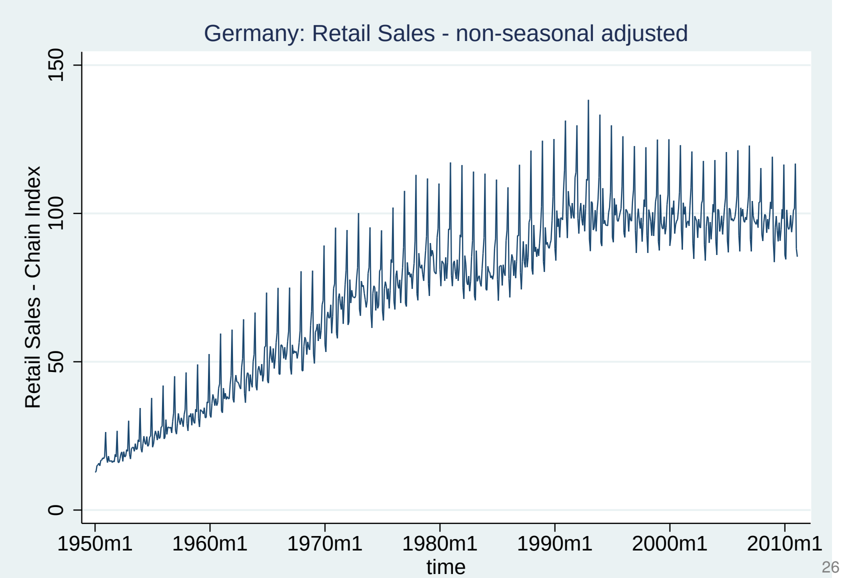
Time Series

- A time series is timely ordered sequence of observations.
- We denote y_t as an observation of a specific variable at date t.
- A time series is list of observations denoted as $\{y_1, y_2, \ldots, y_T\}$ or in short $\{y_t\}_{t=1}^{T}$.
- **What are typical characteristics of times series?**

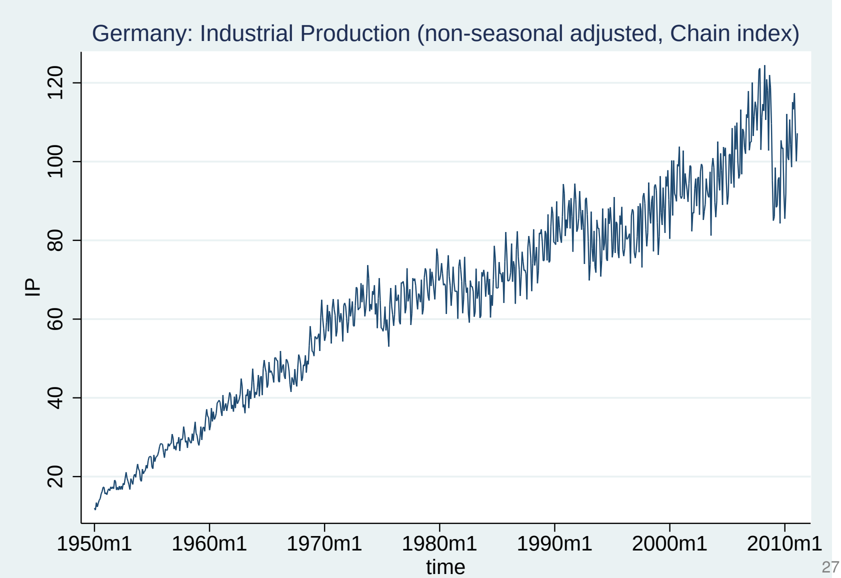
Economic Time Series: GDP I

Economic Time Series: GDP II

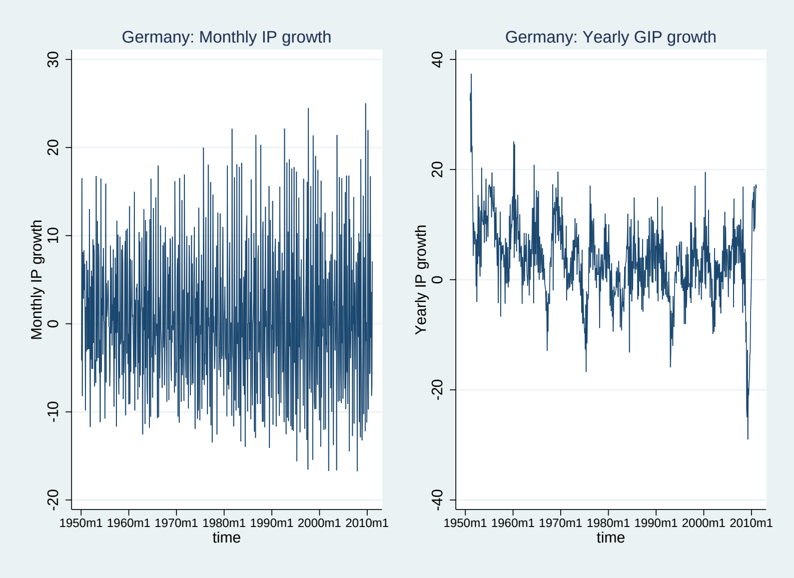
Economic Time Series: Retail Sales



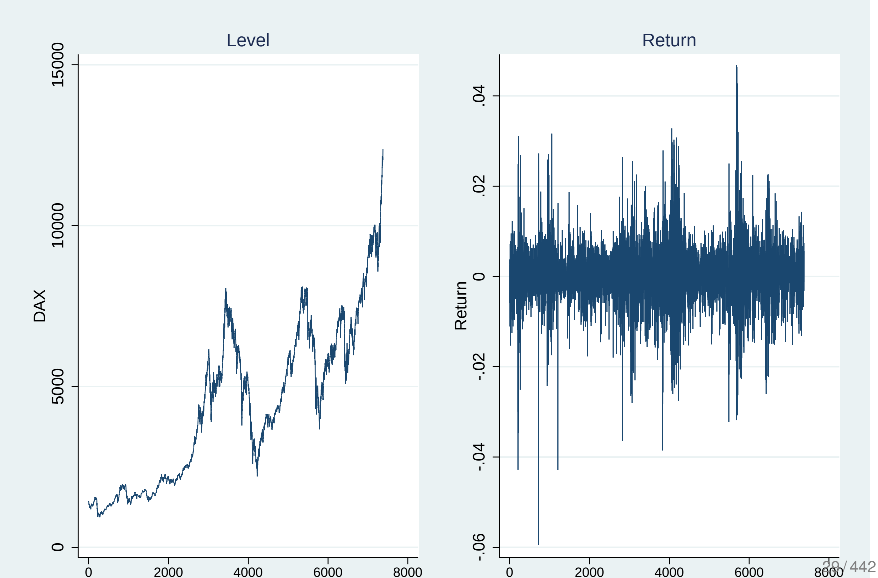
Economic Time Series: Industrial Production



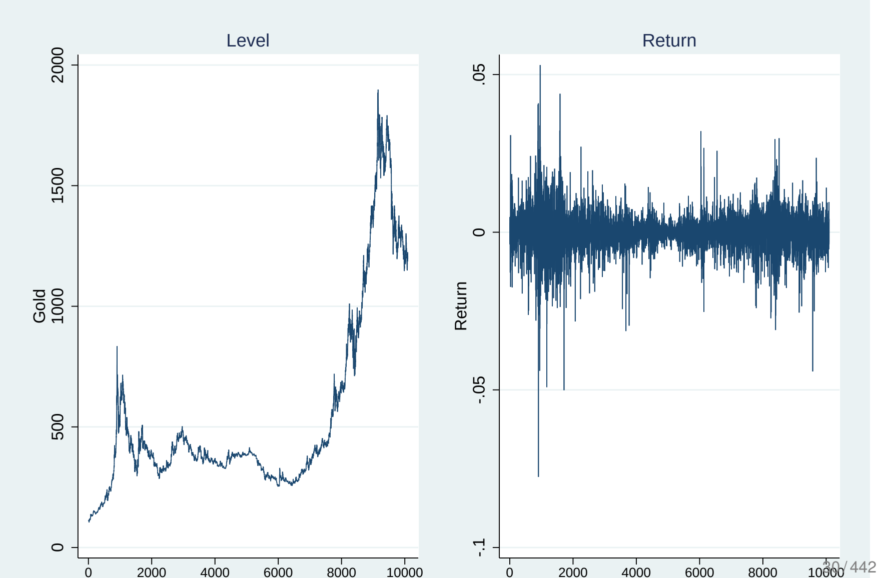
Economic Time Series: Industrial Production



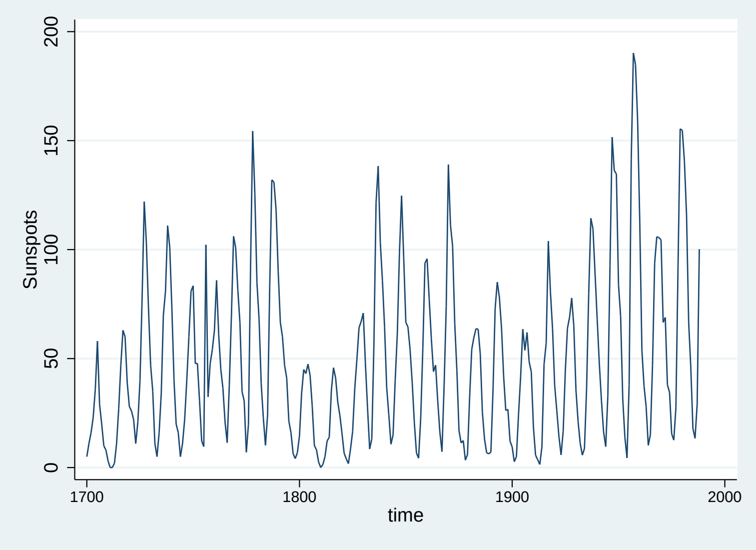
Economic Time Series: The German DAX



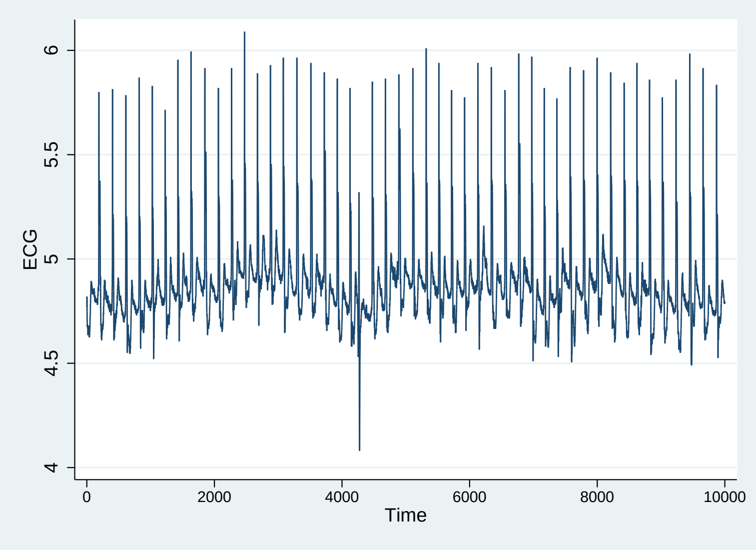
Economic Time Series: Gold Price



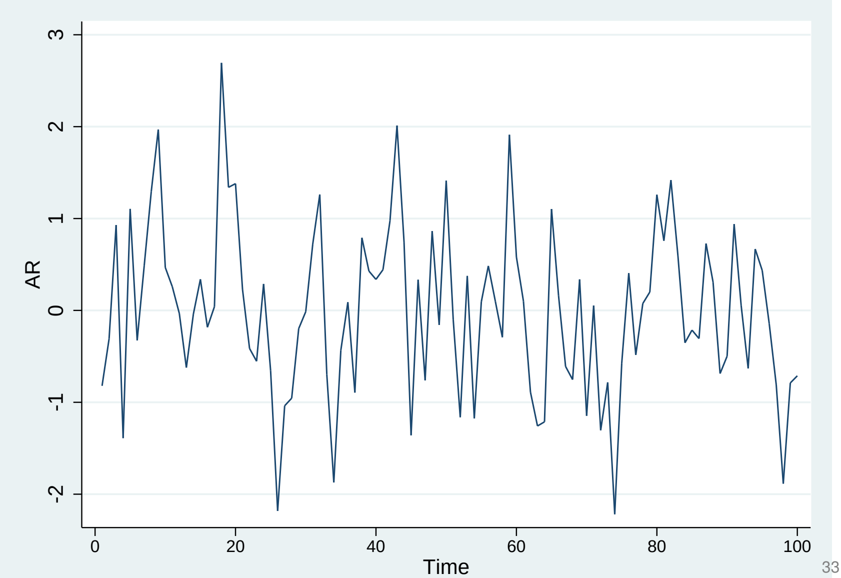
Further Time Series: Sunspots



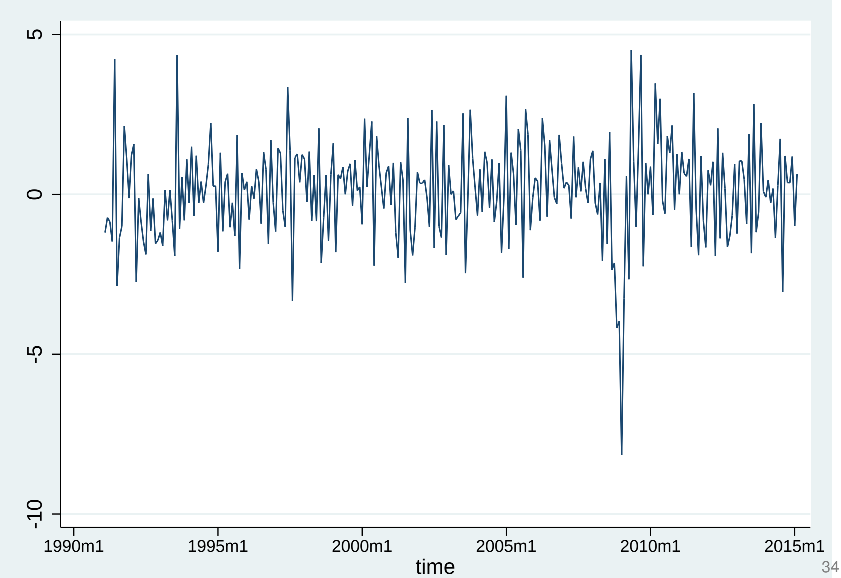
Further Time Series: ECG



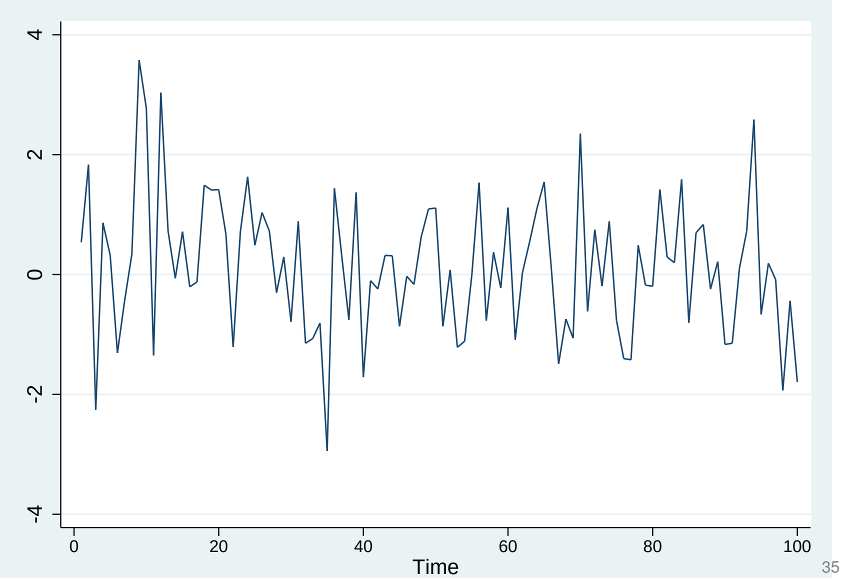
Further Time Series: Simulated Series: AR(1)



Further Time Series: Chaos or a real time series?



Further Time Series: Chaos?



Characteristics of Time series

- Trends
- Periodicity (cyclicality)
- Seasonality
- Volatility Clustering
- Nonlinearities
- Chaos

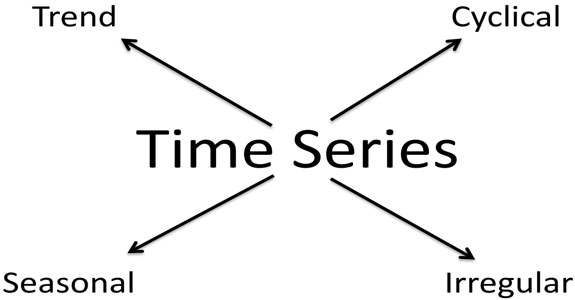
Necessity of (economic) Forecasts

- For political actions and budget control governments need forecasts for macroeconomic variables
  GDP, interest rates, unemployment rate, tax revenues etc.
- marketing need forecasts for sales related variables
  - future sales
  - product demand (price dependent)
  - changes in preferences of consumers

Necessity of (economic) Forecasts

- retail sales company need forecasts to optimize warehousing and employment of staff
- firms need to forecasts cash-flows in order to account of illiquidity phases or insolvency
- university administrations needs forecasts of the number of students for calculation of student fees, staff planning, space organization
- migration flows
- house prices

Time series decomposition



Time series decomposition

Classical additive decomposition:

$$y_t = d_t + c_t + s_t + \epsilon_t \tag{1}$$

- d_t trend component (deterministic, almost constant over time)
- c_t cyclical component (deterministic, periodic, medium term horizons)
- s_t seasonal component (deterministic, periodic; more than one possible)
- ϵ_t irregular component (stochastic, stationary)

Time series decomposition

Goal:

- Extraction of components d_t, c_t and s_t
- The irregular component

$$\epsilon_t = y_t - d_t - c_t - s_t$$

  should be stationary and ideally white noise.
- Main task is then to model the components appropriately.
- Data transformation maybe necessary to account for heteroscedasticity (e.g. log-transformation to stabilize seasonal fluctuations)

Time series decomposition

The multiplicative model:

$$y_t = d_t \cdot c_t \cdot s_t \cdot \epsilon_t \tag{2}$$

will be treated in the tutorial.

Simple Filters

- $$series = signal + noise \tag{3}$$
- The statistician would says
$$series = fit + residual \tag{4}$$
- At a later stage:
$$series = model + errors \tag{5}$$
$\Rightarrow$ mathematical function plus a probability distribution of the error term

Linear Filters

A linear filter converts one times series (x_T) into another (y_t) by the linear operation

$$y_t = \sum_{r=-q}^{+s} a_r x_{t+r}$$

where a_r is a set of weights. In order to smooth local fluctuation one should chose the weight such that

$$\sum a_r = 1$$

The idea

$$y_t = f(t) + \epsilon_t \tag{6}$$

We assume that $f(t)$ and ϵ_t are well-behaved.
Consider N observations at time t_j which are reasonably close in time to t_i. One possible smoother ist

$$y^*_{t_i} = 1/N \sum y_{t_j} = 1/N \sum f(t_j) + 1/N \sum \epsilon_{t_j} \approx f(t_i) + 1/N \sum \epsilon_{t_j} \tag{7}$$

if $\epsilon_t \sim N(0, \sigma^2)$, the variance of the sum of the residuals is σ^2/N^2.
The smoother is characterized by

- span, the number of adjacent points included in the calculation
- type of estimator (median, mean, weighted mean etc.)

Moving Average

- Used for time series smoothing.
- Consists of a series of arithmetic means.
- Result depends on the window size L (number of included periods to calculate the mean).
- In order to smooth the cyclical component, L should exceed the cycle length
- L should be uneven (avoids another cyclical component)

Moving Average

$$
\begin{aligned}
MA(y_t) &= \frac{1}{2q+1} \sum_{r=-q}^{+q} y_{t+r} \\
L &= 2q+1
\end{aligned}
$$

where the weights are given by

$$a_r = \frac{1}{2q+1}$$

Moving Average

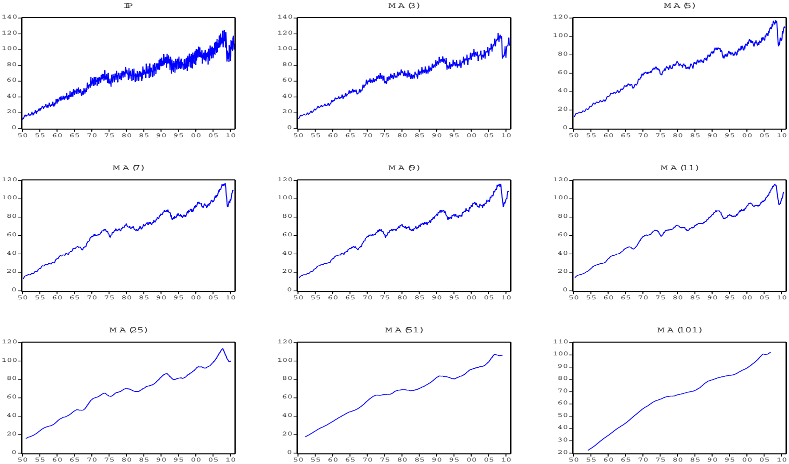
Example: Moving Average (MA) over 3 Periods

- First MA term: $MA_2(3) = \frac{y_1+y_2+y_3}{3}$
- Second MA term: $MA_3(3) = \frac{y_2+y_3+y_4}{3}$

Moving Average

Year	Projects	MA(3) L=3
2005	2	
2006	5	3
2007	2	3
2008	2	3.67
2009	7	5
2010	6	

Moving Average



⇒ the larger L the smoother and shorter the filtered series

EXAMPLE

Generate a random time series (normally distributed) with $T = 20$

- Quick and dirty: Moving Average with Excel
- Nice and Slow: Write a simple Matlab program for calculating a moving average of order L
- Additional Task: Increase the number of observations to $T = 100$, include a linear time trend and calculate different MAs
- Variation: Include some outliers and see how the calculations change.

Exponential Smoothing

- weighted moving averages
- latest observation has the highest weight compared to the previous periods

$$\hat{y}_t = w y_t + (1 - w)\hat{y}_{t-1}$$

Repeated substitution gives:

$$\hat{y}_t = w \sum_{s=0}^{t-1} (1 - w)^s y_{t-s}$$

$\Rightarrow$ that's why it is called exponential smoothing, forecasts are the weighted average of past observations where the weights decline exponentially with time.

Exponential Smoothing

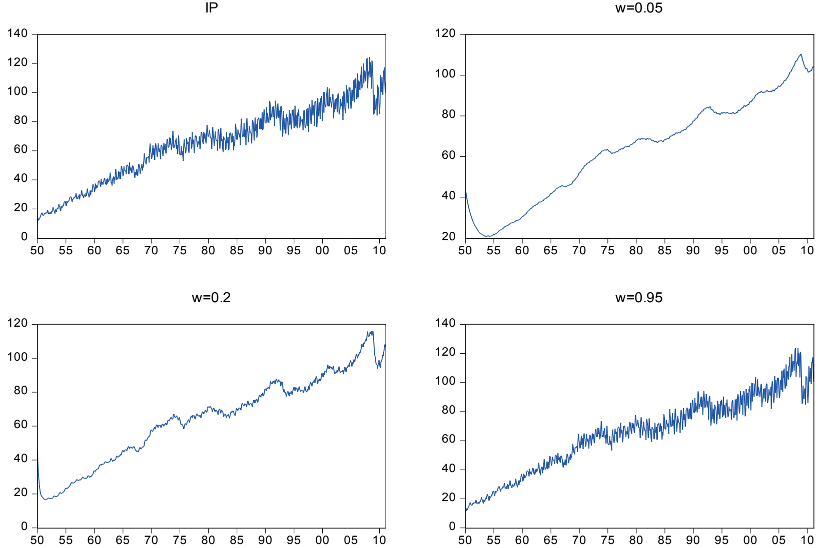
- Is used for smoothing and short–term forecasting
- Choice of w:
  - subjective or through calibration
  - numbers between 0 and 1
    Close to 0 for smoothing out unpleasant cyclical or irregular components
    Close to 1 for forecasting

Exponential Smoothing

$$\hat{y}_t = wy_t + (1-w)\hat{y}_{t-1} \quad w = 0.2$$

Year	Projects	Smothed Value	Forecast
2005	2	2	-
2006	5	0.2*5+0.8*2 =2.6	2.000
2007	2	0.2*2+0.8*2.6 =2.48	2.600
2008	2	0.2*2+0.8*2.48 =2.3684	2.480
2009	7	0.2*7+0.8*2.384=3.307	2.384
2010	6	0.2*6+0.8*3.307 =3.846	3.307

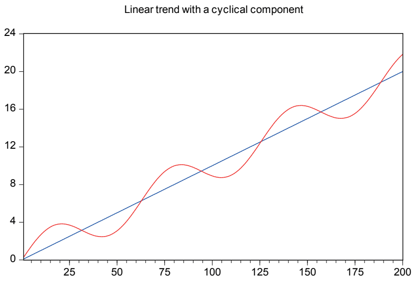
Exponential Smoothing

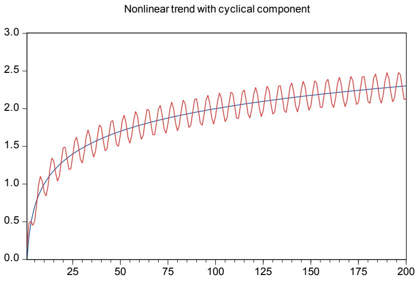


Trend Component

- positive or negative trend
- observed over a longer time horizon
- linear vs. non–linear trend
- smooth vs. non–smooth trends
- $\Rightarrow$ trend is 'unobserved' in reality

Trend Component: Example





Why is trend extraction so important?

The case of detrending GDP

- trend GDP is denoted as **potential output**
- The difference between trend and actual GDP is called the **output gap**
- Is an economy below or above the current trend? (Or is the output gap positive or negative?)
  $\Rightarrow$ consequences for economic policy (wages, prices etc.)
- Trend extraction can be highly controversial!

Linear Trend Model

Year	Time (x_t)	Turnover (y_t)
05	1	2
06	2	5
07	3	2
08	4	2
09	5	7
10	6	6

$$y_t = \alpha + \beta x_t$$

Linear Trend Model

Estimation with OLS

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t = 1.4 + 0.743 x_t$$

Forecast for 2011:

$$\hat{y}_{2011} = 1.4 + 0.743 \cdot 7 = 6.6$$

Quadratic Trend Model

Year	Time (x_t)	Time^2 (x_t^2)	Turnover (y_t)
05	1	1	2
06	2	4	5
07	3	9	2
08	4	16	2
09	5	25	7
10	6	36	6

$$y_t = \alpha + \beta_1 x_t + \beta_2 x_t^2$$

Quadratic Trend Model

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t + \hat{\beta}_2 x_t^2 = 3.4 - 0.757143 x_t + 0.214286 x_t^2$$

Forecast for 2011:

$$\hat{y}_{2011} = 3.4 - 0.757143 \cdot 7 + 0.214286 \cdot 7^2 = 8.6$$

Exponential Trend Model

Year	Time (x_t)	Turnover (y_t)
05	1	2
06	2	5
07	3	2
08	4	2
09	5	7
10	6	6

$$y_t = \alpha \beta_1^{x_t}$$

$\Rightarrow$ Non-linear Least Squares (NLS) or
Linearize the model and use OLS:

$$\log y_t = \log \alpha + \log(\beta_1) x_t$$

$\Rightarrow$ 'relog' the model

Exponential Trend Model

Estimation via **NLS**:

$$\hat{y}_t = \hat{\alpha} + \hat{\beta}_1^{\,x_t} = 0.08 \cdot 1.93^{x_t}$$

Forecast for 2011:

$$\hat{y}_{2011} = 0.08 \cdot 1.93^7 = 15.4$$

Logarithmic Trend Model

Year	Time (x_t)	log(*Time*)	Turnover (y_t)
05	1	log(1)	2
06	2	log(2)	5
07	3	log(3)	2
08	4	log(4)	2
09	5	log(5)	7
10	6	log(6)	6

Logarithmic Trend:

$$y_t = \alpha + \beta_1 \log x_t$$

Logarithmic Trend Model

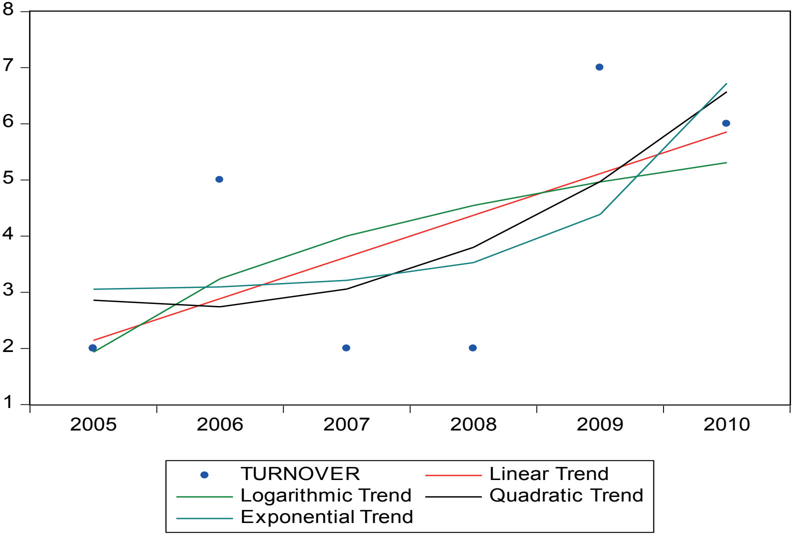
Estimation via **OLS**:

$$\hat{y}_t = \hat{\alpha} + \hat{\beta}_1 \log x_t = 1.934675 + 1.883489 \cdot \log y_t$$

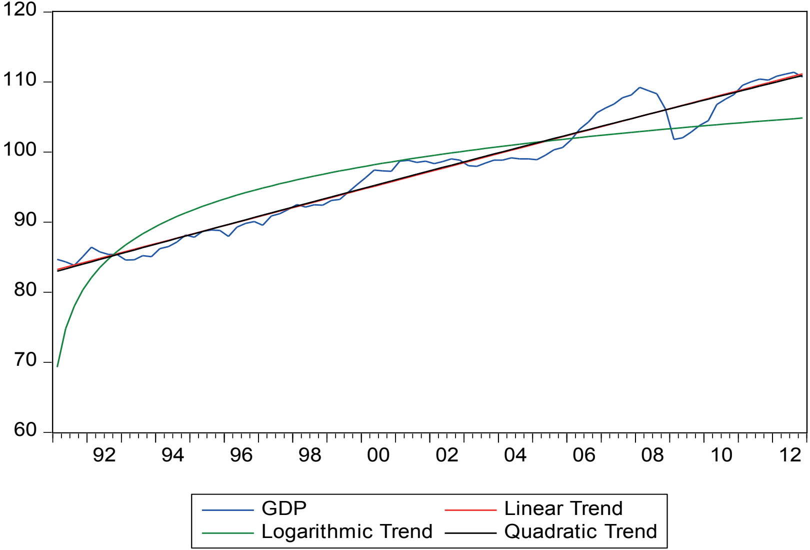
Forecast for 2011:

$$\hat{Y}_{2011} = 1.934675 + 1.883489 \cdot \log(7) = 5.6$$

Comparison of different trend models



Detrending GDP



Which trend model to choose?

- Linear Trend model, if the first differences

$$y_t - y_{t-1}$$

  are stationary
- Quadratic trend model, if the second differences

$$(y_t - y_{t-1}) - (y_{t-1} - y_{t-2})$$

  are stationary
- Logarithmic trend model, if the relative differences

$$\frac{y_t - y_{t-1}}{y_t}$$

  are stationary

The Hodrick-Prescott-Filter (HP)

The HP extracts a flexible trend. The filter is given by

$$\min_{\mu_t} \sum_{t=1}^{T} [(y_t - \mu_t)^2 + \lambda \sum_{t=2}^{T-1} \{(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})\}^2] \qquad (8)$$

where μ_t is the flexible trend and λ a smoothness parameter chosen by the researcher.

- As λ approaches infinity we obtain a linear trend.
- Currently the most popular filter in economics.

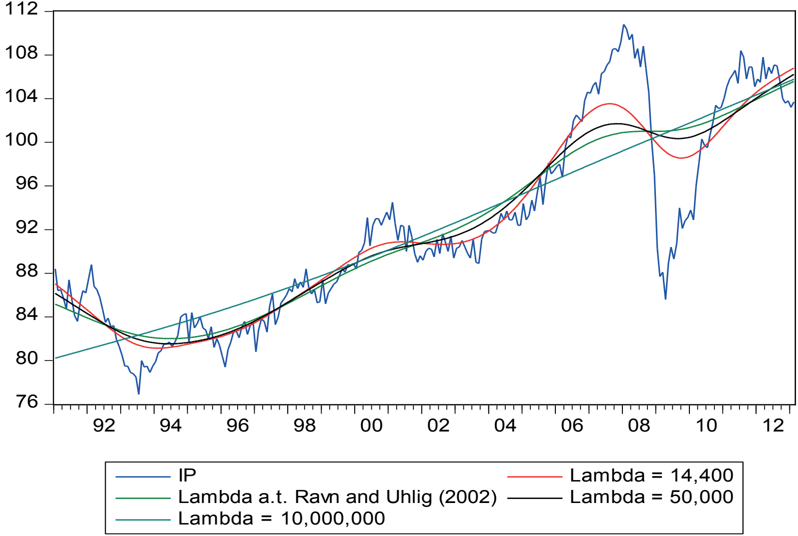
The Hodrick-Prescott-Filter (HP)

How to choose λ?
Hodrick-Prescot (1997) recommend:

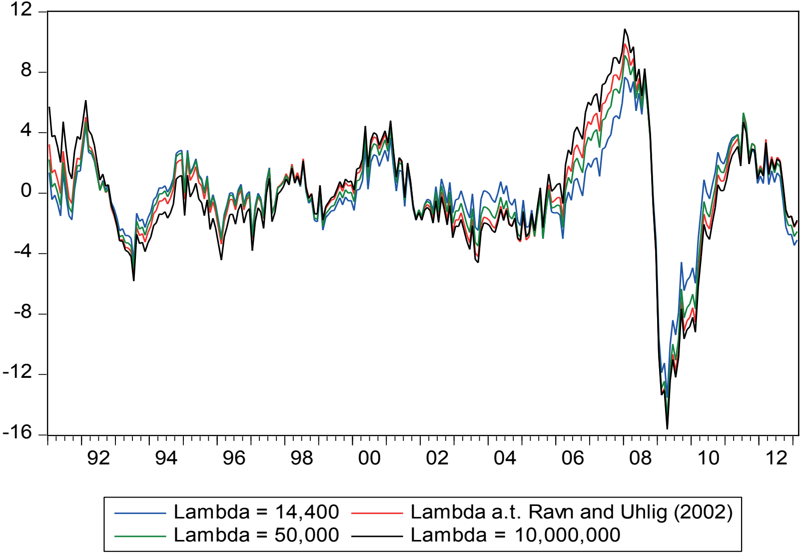
$$\lambda = \begin{cases} 100 \text{ for annual data} \\ 1600 \text{ for quarterly data} \\ 14400 \text{ for monthly data} \end{cases} \tag{9}$$

Alternative: Ravn and Uhlig (2002)

The Hodrick-Prescott-Filter (HP)



The Hodrick-Prescott-Filter (HP)

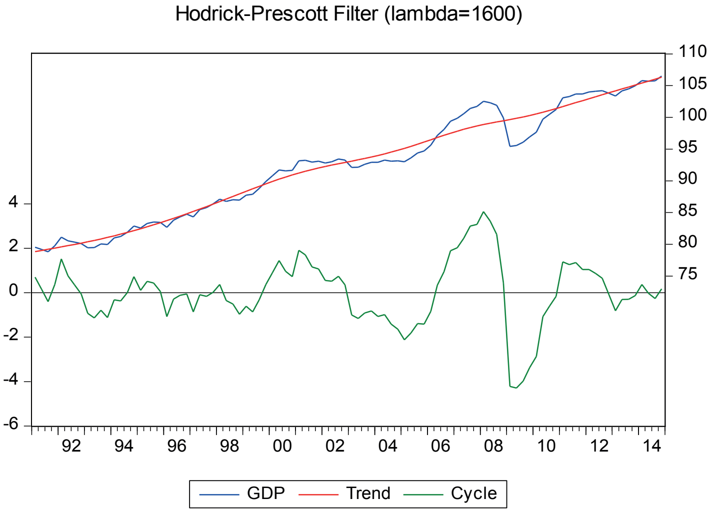


Problems with the HP-Filter

- λ is a 'tuning' parameter
- end of sample instability
  $\Rightarrow$ AR-forecasts

Case study for German GDP: Where are we now?

HP-Filter


Hodrick-Prescott Filter (lambda=1600)
110
105
100
95
90
85
80
75
4
2
0
-2
-4
-6
92
94
96
98
00
02
04
06
08
10
12
14
GDP
Trend
Cycle

Can we test for a trend?

- Yes and no
- If the trend component significant?
- several trends can be significant
- Trend might be spurious
- Is it plausible that there is a trend?
- A priori information by the researcher
- unit roots

EXAMPLE

Time series: Industrial Production in Germany (1991:01-2014:02)

- Plot the time series and state which trend adjustment might be appropriate
- Prepare your data set in Excel and estimate various trends in Eviews
- Which trend would you choose?

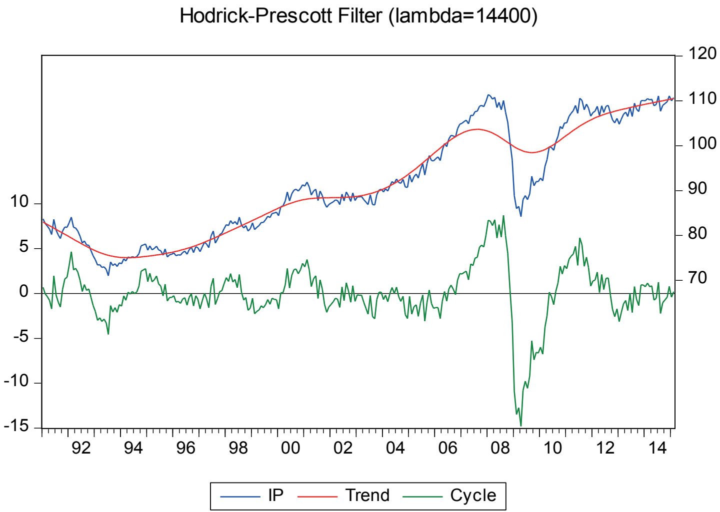
Cyclical Component

- is not always present in time series
- Is the difference between the observed time series and the estimated trend

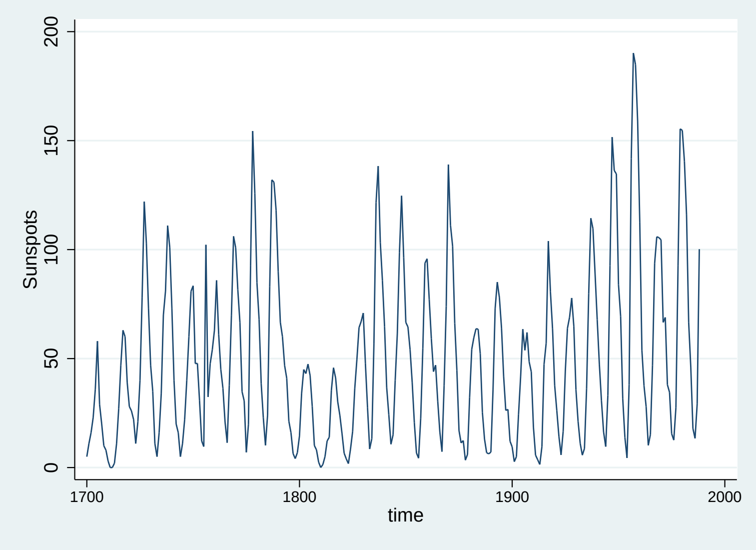
In economics

- characterizes the Business cycle
- different length of cycles (3-5 or 10-15 years)

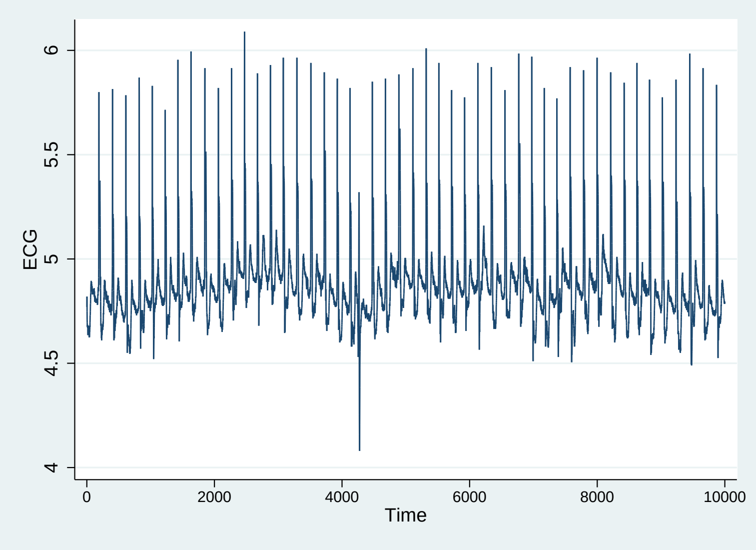
Cyclical Component: Example



Cyclical Component: Example II



Cyclical Component: Example III

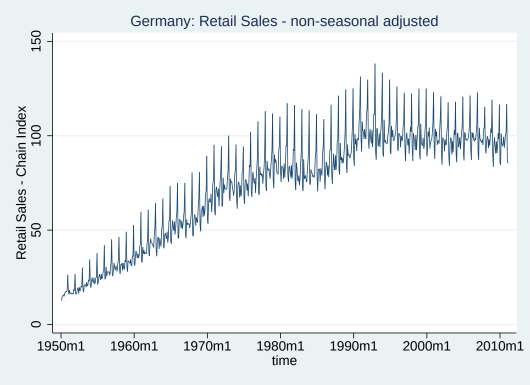


Can we test for a cyclical component?

- Yes and no
- see the trend section
- Does a cycle make sense?

Seasonal Component

- similar upswings and downswings in a fixed time interval
- regular pattern, i.e. over a year



Types of Seasonality

- A: $y_t = m_t + S_t + \epsilon_t$
- B: $y_t = m_t S_t + \epsilon_t$
- C: $y_t = m_t S_t \epsilon_t$

Model A is additive seasonal, Models B and C contains multiplicative seasonal variation

Types of Seasonality

- if the seasonal effect is constant over the seasonal periods
  $\Rightarrow$ additive seasonality (Model A)
- if the seasonal effect is proportional to the mean
  $\Rightarrow$ multiplicative seasonality (Model A and B)
- in case of multiplicative seasonal models use the logarithmic transformation to make the effect additive

Seasonal Adjustment

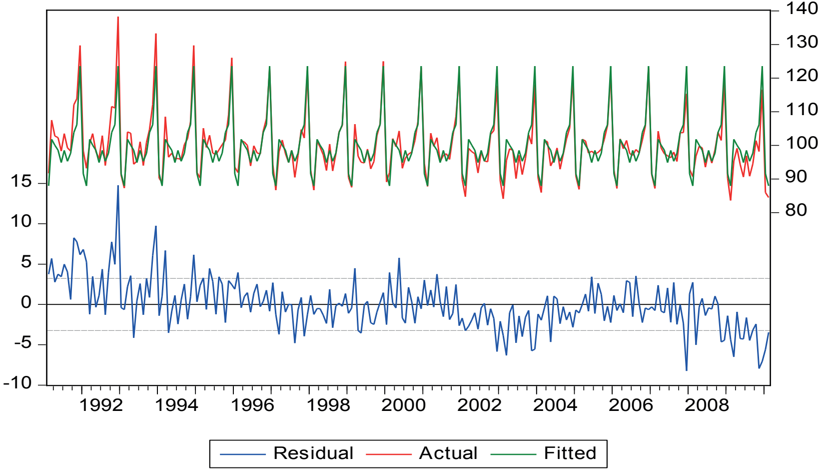
Simplest Approach to seasonal adjustment:

- Run the time series on a set of dummies without a constant (Assumes that the seasonal pattern is constant over time)
- the residuals of this regression are seasonal adjusted
- Example: Monthly data

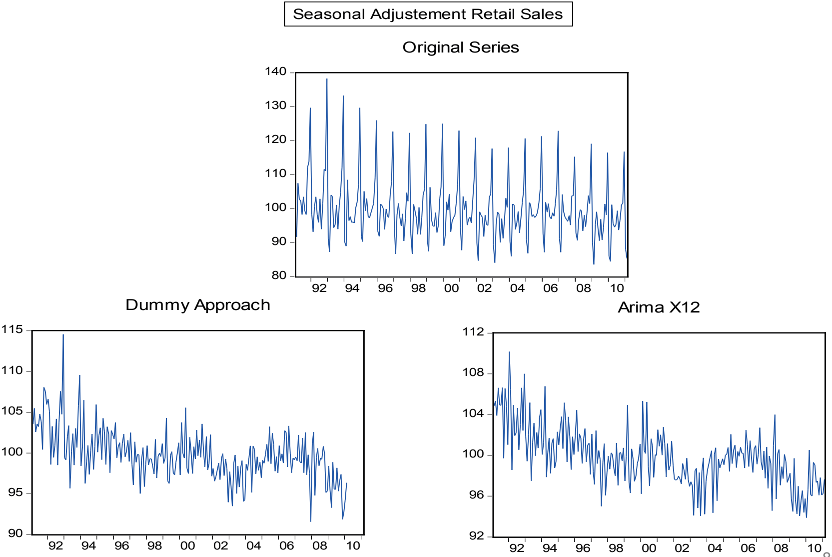
$$
\begin{aligned}
y_t &= \sum_{i=1}^{12} \beta_i D_i + \epsilon_t \\
\epsilon_t &= y_t - \sum_{i=1}^{12} \hat{\beta} D_i \\
y_{t,sa} &= \epsilon_t + mean(y_t)
\end{aligned}
$$

- The most well known seasonal adjustment procedure: CENSUS X12 ARIMA

Seasonal Adjustment: Dummy Regression Example



Seasonal Adjustment: Example



Seasonal Moving Averages

For monthly data one can employ the filter

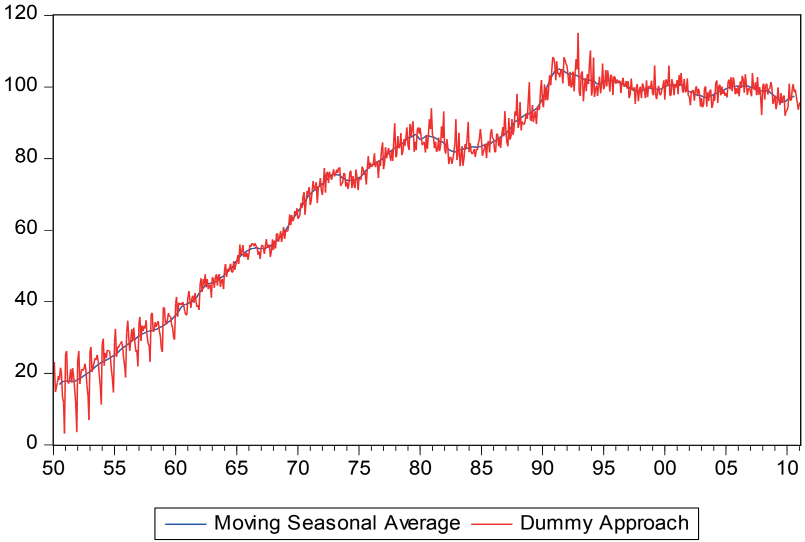
$$SMA(y_t) = \frac{\frac{1}{2}y_{t-6} + y_{t-5} + y_{t-4} + \ldots + y_{t+6} + \frac{1}{2}y_{t+6}}{12}$$

or for quarterly data

$$SMA(y_t) = \frac{\frac{1}{2}y_{t-2} + y_{t-1} + y_t + y_{t+1} + \frac{1}{2}y_{t+2}}{4}$$

- Note: The weights add up to one!
- Standard moving average not applicable

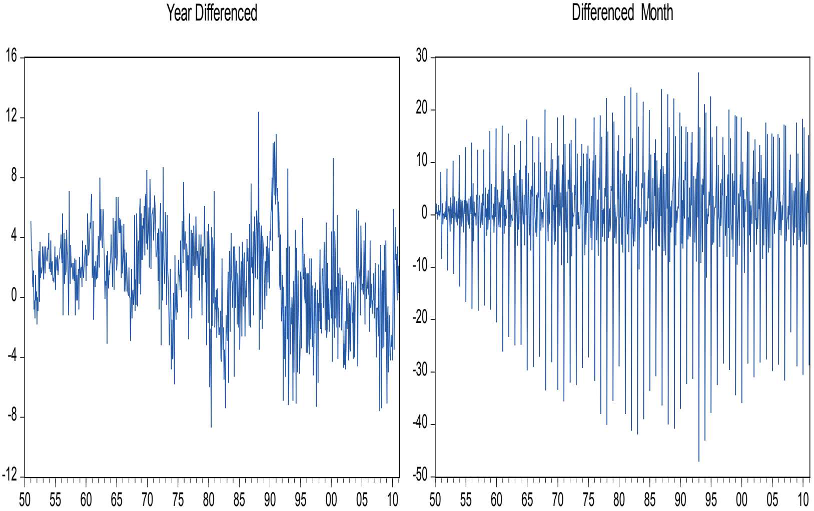
Seasonal Moving Averages: Retail Sales Example



Seasonal Differencing

- seasonal effect can be eliminated using the a simple linear filter
- in case of a monthly time series: $\Delta_{12} y_t = y_t - y_{t-12}$
- in case of a quarterly time series: $\Delta_4 y_t = y_t - y_{t-4}$

Seasonal Differencing: Retail Sales Example



Can we test for seasonality?

- Yes and no
- Does seasonality makes sense?
- Compare the seasonal adjusted and unadjusted series
- look into the ARIMA X12 output
- Be aware of changing seasonal patterns

EXAMPLE

Time series: seasonally unadjusted Industrial Production in Germany (1950:01-2011:02)

- Remove the seasonality by a moving seasonal filter
- Try the dummy approach
- Finally, use the ARIMAX12-Approach
- Start the sample in 1991:01 and compare all filters with the full sample

Irregular Component

- erratic, non-systematic, random "residual" fluctuations due to random shocks
  - in nature
  - due to human behavior
- no observable iterations

Can we test for an irregular component?

- YES
- several tests available whether the irregular component is a white noise or not

White Noise

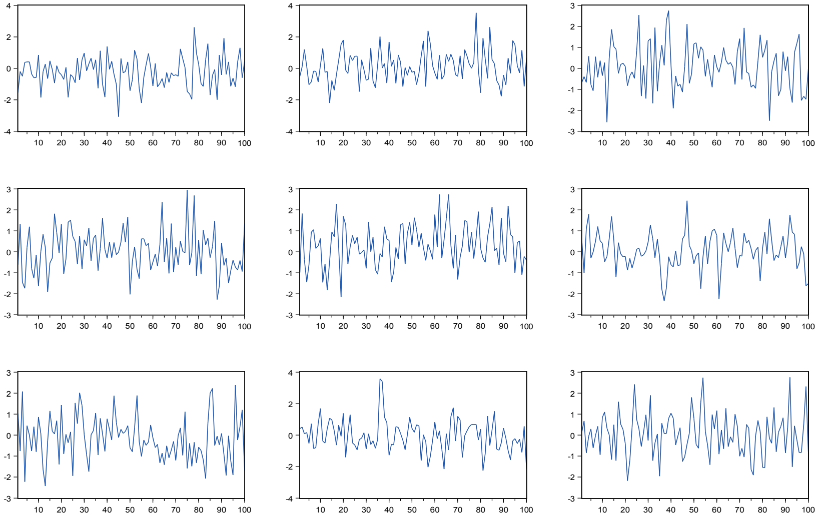
A process $\{y_t\}$ is called **white noise** if

$$\begin{aligned} \mathrm{E}(y_t) &= 0 \\ \gamma(0) &= \sigma^2 \\ \gamma(h) &= 0 \text{ for } |h| > 0 \end{aligned}$$

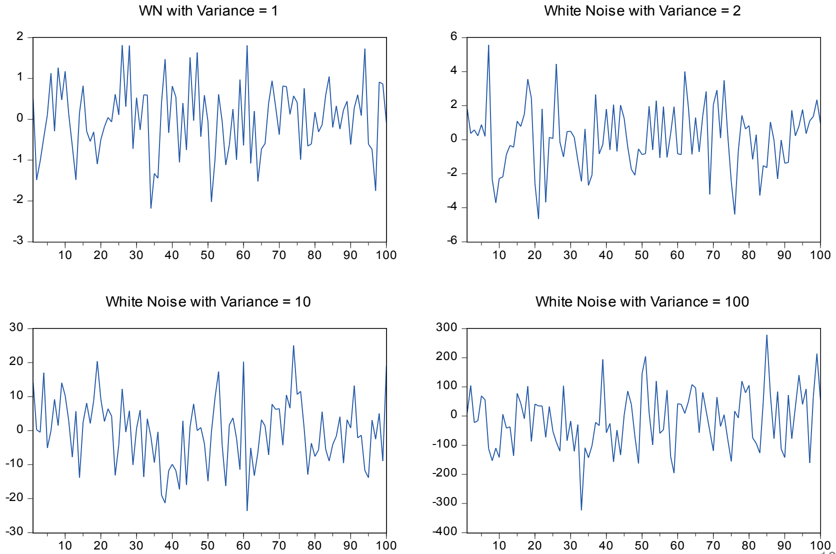
$\Rightarrow$ all y_t's are uncorrelated. We write: $\{y_t\} \sim WN(0, \sigma^2)$

White Noise

White Noise with Variance = 1



White Noise


WN with Variance = 1
White Noise with Variance = 2
White Noise with Variance = 10
White Noise with Variance = 100

Random Walk (with drift)

A simple *random walk* is given by

$$y_t = y_{t-1} + \epsilon_t$$

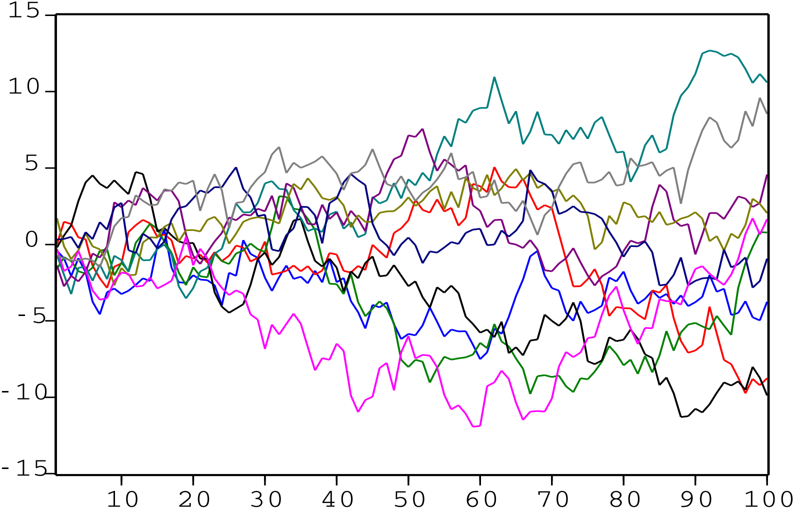
By adding a constant term

$$y_t = c + y_{t-1} + \epsilon_t$$

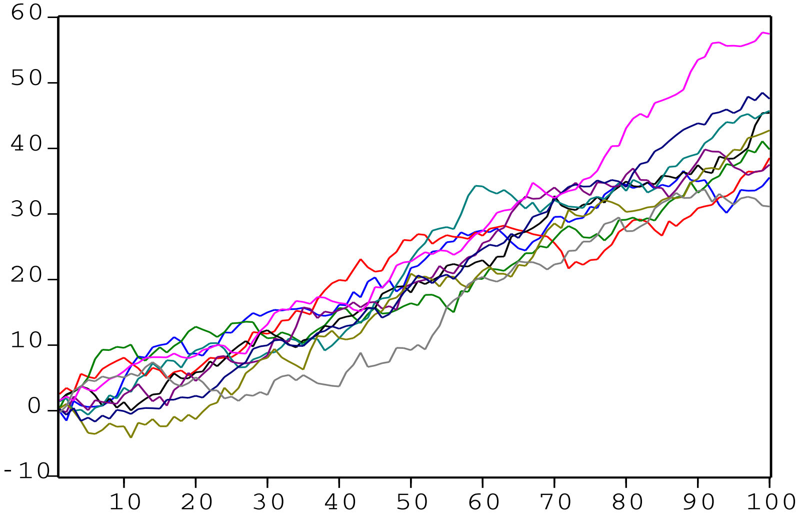
we get a *random walk with drift*. It follows that

$$y_t = ct + \sum_{j=1}^{t} \epsilon_j$$

Random Walk: Examples



Random Walk with Drift: Examples



EXAMPLE

Fun with Random Walks

- Generate 50 different random walks
- Plot all random walks
- Try different variances and distributions

Autoregressive processes

- especially suitable for (short-run) forecasts
- utilizes autocorrelations of lower order
  - 1st order: correlations of successive observations
  - 2nd order: correlations of observations with two periods in between
- Autoregressive model of order p

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + \epsilon_t$$

Autoregressive processes

Number of machines produced by a firm

Year	Units
2003	4
2004	3
2005	2
2006	3
2007	2
2008	2
2009	4
2010	6

$\Rightarrow$ Estimation of an AR model of order 2

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \epsilon_t$$

Autoregressive processes

Estimation Table:

Year	Constant	y_t	y_{t-1}	y_{t-2}
2003	1	4		
2004	1	3	4	
2005	1	2	3	4
2006	1	3	2	3
2007	1	2	3	2
2008	1	2	2	3
2009	1	4	2	2
2010	1	6	4	2

$\Rightarrow$ OLS

$$\hat{y}_t = 3.5 + 0.8125 y_{t-1} - 0.9375 y_{t-2}$$

Autoregressive processes

Forecasting with an AR(2) model:

$$\begin{aligned}
\hat{y}_t &= 3.5 + 0.8125 y_{t-1} - 0.9375 y_{t-2} \\
y_{2011} &= 3.5 + 0.8125 y_{2010} - 0.9375 y_{2009} \\
&= 3.5 + 0.8125 \cdot 6 - 0.9375 \cdot 4 \\
&= 4.625
\end{aligned}$$